

Binomial Theorem Practice Worksheet

Instructions

- This worksheet covers concepts from the Binomial Theorem.
- Answer all questions.
- Refer to your textbook for definitions and formulas if needed.

Section A: Multiple Choice Questions (MCQ)

1. What is the total number of terms in the expansion of $(a+b)^n$?
 - n
 - $n-1$
 - $n+1$
 - $2n$
2. In the expansion of $(a+b)^n$, how do the powers of the first quantity ' a ' change?
 - They increase by 1 in successive terms.
 - They decrease by 1 in successive terms.
 - They remain constant.
 - They vary randomly.
3. The coefficients ${}^nC_k$ in the binomial theorem are also known as:
 - Exponential coefficients
 - Pascal's coefficients
 - Algebraic coefficients
 - Series coefficients
4. What is the sum of the indices of a and b in every term of the expansion of $(a+b)^n$?
 - $n-1$
 - n
 - $n+1$
 - $2n$
5. The expansion of $(1+x)^n$ is given by:

- ${}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_n x^n$
- ${}^nC_0 x^n + {}^nC_1 x^{n-1} + \dots + {}^nC_n$
- $(1-x)^n$
- $(x+1)^n$

6. The value of $(a+b)^0$ is:

- 0
- 1
- a+b
- undefined

7. Pascal's Triangle is used to arrange:

- terms of expansion
- powers of variables
- binomial coefficients
- algebraic expressions

8. Which of the following is equivalent to ${}^nC_r$?

- ${}^nC_{n-r}$
- ${}^nC_{r+1}$
- ${}^nC_{r-1}$
- ${}^{n+1}C_r$

9. What is the coefficient of $a^2 b^2$ in the expansion of $(a+b)^4$?

- 4
- 6
- 1
- 10

10. In the expansion of $(x-y)^n$, what is the sign of the terms involving odd powers of y ?

- Always positive
- Always negative
- Alternating, starting positive
- Alternating, starting negative

Section B: Short Answer Questions

1. Expand $(1-2x)^5$ using the Binomial Theorem.
2. Find the value of $(101)^4$ using the binomial theorem.

- Expand $(2x - x/2)^5$.
- Expand $(2x - 3)^6$.
- Using binomial theorem, evaluate $(99)^3$.
- Prove that $9^{n+1} - 8n - 9$ is divisible by 64, whenever n is a positive integer.
- Find the value of $((3) + (2))^6 - ((3) - (2))^6$.
- Find the value of $((3) + 1)^4 - ((3) - 1)^4$.
- Using binomial theorem, evaluate $(102)^5$.
- Show that $(1.01)^{1000000}$ is greater than 10000.

Answer Key

Section A: Multiple Choice Questions

- $n+1$
- They decrease by 1 in successive terms.
- Pascal's coefficients
- n
- ${}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_n x^n$
- 1
- binomial coefficients
- ${}^nC_{n-r}$
- 6
- Alternating, starting negative

Section B: Short Answer Questions

- $1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$
- 104060401
- $(32)/(x^5) - (40)/(x^3) + 20/x - 5x + 5/8x^3 - (x^5)/(32)$
- $64x^6 - 576x^5 + 2160x^4 - 4320x^3 + 4860x^2 - 2916x + 729$
- 970299

- Let the given expression be $E = 9^{n+1} - 8n - 9$.

We can write 9^{n+1} as $(1+8)^{n+1}$.

Using the binomial theorem, $(1+8)^{n+1} = {}^{n+1}C_0 8^0 + {}^{n+1}C_1 8^1 + {}^{n+1}C_2 8^2 + {}^{n+1}C_3 8^3 + \dots + {}^{n+1}C_n 8^n + {}^{n+1}C_{n+1} 8^{n+1}$.

$$= 1 + (n+1)8 + {}^{n+1}C_2 8^2 + {}^{n+1}C_3 8^3 + \dots + 8^{n+1}$$

$$= 1 + 8n + 8 + 64[{}^{n+1}C_2 + {}^{n+1}C_3 + \dots + 8^{n-1}]$$

$$= 9 + 8n + 64K, \text{ where } K = {}^{n+1}C_2 + {}^{n+1}C_3 + \dots + 8^{n-1} \text{ is an integer.}$$

Substituting this back into the expression for E :

$$E = (9 + 8n + 64K) - 8n - 9 = 64K.$$

Since K is an integer, E is divisible by 64.

$$\begin{aligned} 7. \quad & 2[61((3))^5((2))^1 + 63((3))^3((2))^3 + 65((3))^1((2))^5] \\ &= 2[6(9(3))((2)) + 20(3(3))(2(2)) + 6((3))(4(2))] \\ &= 2[54(6) + 120(6) + 24(6)] \\ &= 2[198(6)] = 396(6) \end{aligned}$$

$$\begin{aligned} 8. \quad & 2[41((3))^3(1)^1 + 43((3))^1(1)^3] \\ &= 2[4(3(3)) + 4((3))] \\ &= 2[12(3) + 4(3)] \\ &= 2[16(3)] = 32(3) \end{aligned}$$

$$9. \quad 11040808032$$

$$10. \text{ Consider } (1.01)^{1000000} = (1 + 0.01)^{1000000}.$$

Using the binomial expansion:

$$\begin{aligned} (1 + 0.01)^{1000000} &= 1000000(1)^{1000000}(0.01)^0 + 1000000(1)^{999999}(0.01)^1 + 1000000(1)^{999998}(0.01)^2 + \\ &= 1 + 1000000 \times 0.01 + \text{positive terms} \\ &= 1 + 10000 + \text{positive terms} \\ &= 10001 + \text{positive terms} \end{aligned}$$

Since the remaining terms are positive, $(1.01)^{1000000} > 10001$.

Therefore, $(1.01)^{1000000}$ is greater than 10000.